Building Charging Stations for Electric Vehicles
When and Where?

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Source: Chevy Volt Wikipedia

A Reality

- Chevy Volt: 3,196 Sold in the US
- Nissan Leaf: 5,000 Sold in the US
- Tesla Roadster
- Conversion Kits
- Planned for market in 2011 or 2012: Fisker Karma, Toyota Prius Plug-in Hybrid, Ford Escape Plug-in Hybrid, Volvo V70 Plug-in Hybrid, Suzuki Swift Plug-in and the Ford C-Max Energi.

Technologies

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- PHEV
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  - Issue: increasing demand
- Renewable Energy Sources
  - Environmentally friendly
  - Issue: intermittent power generation

## Battery Swap Stations

- Why?
    - Not everyone has a place to charge at home
    - Fast battery exchange
    - Long trips
    - The spare batteries at swap stations could participate in vehicle to grid storage

V2G Systems

- V2G: Vehicle-to-Grid
- Give the power from electric vehicles back to the power grid
- Can help overcome the problems by providing control and interaction to renewables and PHEVs
- Focus on a V2G system where PHEV charging stations (not the vehicles themselves) can send power to the power grid

Grid with Charging Stations and Demands
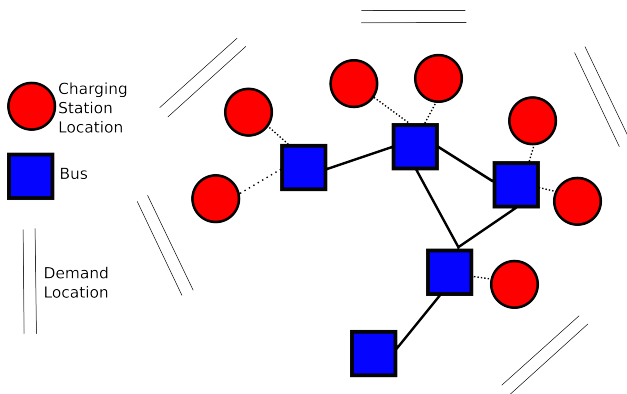
Grid with Charging Stations and Demands
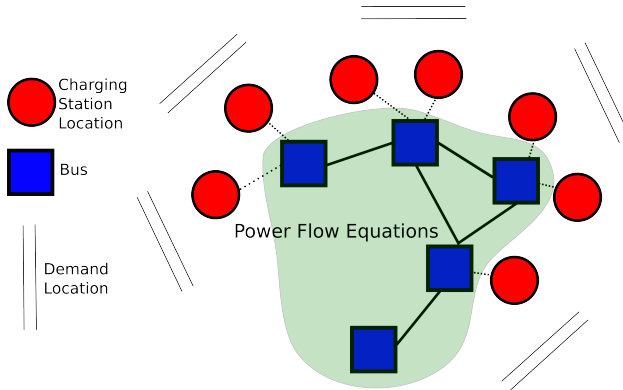
Players and Objectives

- Charging Station Operators: Minimize building costs, maximize revenues
- PHEV drivers: Drive shortest distance to charging stations
- Power Grid Operators: Minimize load shedding costs, generation costs
- Social Welfare: Increase number of PHEVs

Previous Work

The objective is to minimze the sum of all of costs from the previous slide

This objective was formulated in a recent paper, *Locating PHEV Exchange Stations in V2G* by Feng Pan, Russell Bent, et. al. 2010.

- Two-stage location model was developed to site the PHEV exchange stations

- Showed the importance of properly locating stations both from the perspective of the transportation system and the electric power grid.

Some open questions remained, and those are addressed in this presentation.

A Mixed Linear Integer Programming Formulation

Variables: x_i - binary variable which decides whether to build a charging station at location i

$$\min \sum_{i \in I} f_i x_i + \sum_{i \in I, j \in J} c_{ij} y_{ijt} + \sum_{j \in J} h q_{jt} + \sum_{u \in N} o_u \beta_{ut} + \sum_{u \in N} g \delta_{ut}$$
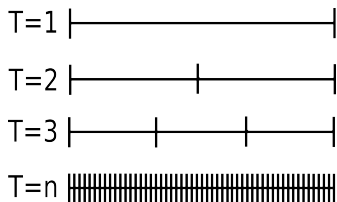
Constraints:

- Capacity
- Demand
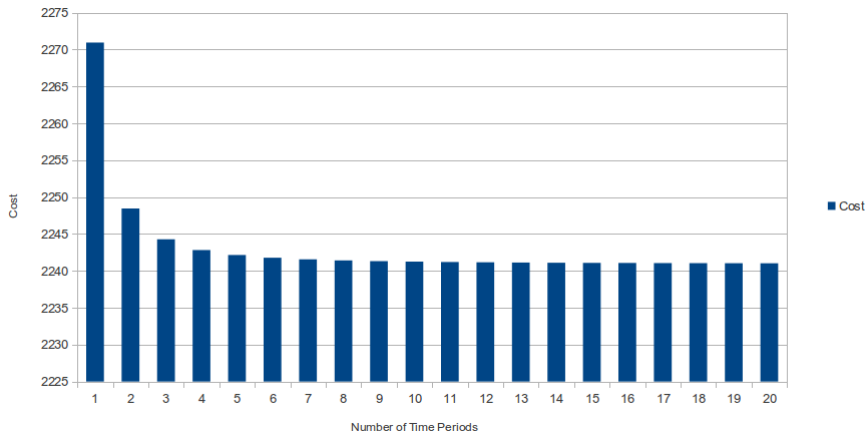- Linearized power flow equations

Open Questions

- Number and locations of those charging stations needs to evolve over time
- Construction of new stations has an impact on future demands
- Weights of costs unclear
- Conflicting objectives: Minimizing the total driving distance has a negative impact on the total number of PHEVs

Add time dimension to the Integer Program

- Need a 'discount factor' to discourage building stations too early

Cost with Different Refinement Levels

Integer Program Issues

- With the proposed integer programming formulation, only simple demand models can be used:
  - Arbitrary demand function, if it does not depend on previous decisions
  - Constraints of the following form: $d_t = a + b d_{t-1} + c \sum_{i \in N} x_i$
  - No fancy relationship, given the nature of a linearly constrained integer problem
- More refinement increases the formulation size
- Still have the problem of having conflicting objectives

Open Questions - What got Solved?

- ✓ Number and locations of those charging stations needs to evolve over time
- ✓ Construction of new stations has an impact on future demands
- Weights of costs unclear
- Conflicting objectives: Minimizing the total driving distance has a negative impact on the total number of PHEVs

Conflicting Objectives Issue

- In the objective function, $\sum_{i \in I, j \in J} c_{ij} y_{ijt}$ tries to minimize the total driving distance
- The only reason to include this is so that demands get assigned to stations that are close by
- But it minimizes an increasing function of demand!

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- But it minimizes an increasing function of demand!
- Bi-Level Programming?
  - $\max_x \{ f(x) + \min_x \{\text{Driving Costs}\} \}$

Continuous Time Formulation

- x_i Time at which to open station i
- Maximize profit of station operators, while considering the interests of other parties
- $\max_x\{\text{revenue from demand and power grid} - \sum_{i \in I} f_i \cdot r^{x_i}\}$
- Requires a completely different approach to solving

Formulation Details

- Demand can be in any analytic form - not as limited as the integer programming formulation
- A challenge is calulating the objective value for given values of x
- Another one is figuring out an efficient way of acutally minimizing that function, it is a smooth unconstrained, but nonconvex funciton.

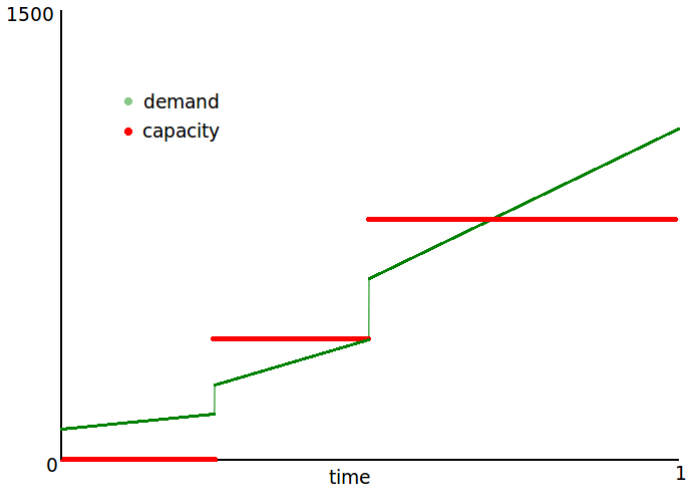## Evaluating the Objective Function: Example

Two charging station locations, one single route, and one bus with no load.

$d(t) = 100 + t(200 + 50 \sum_{i \in N_j} I(x_i < t))$.
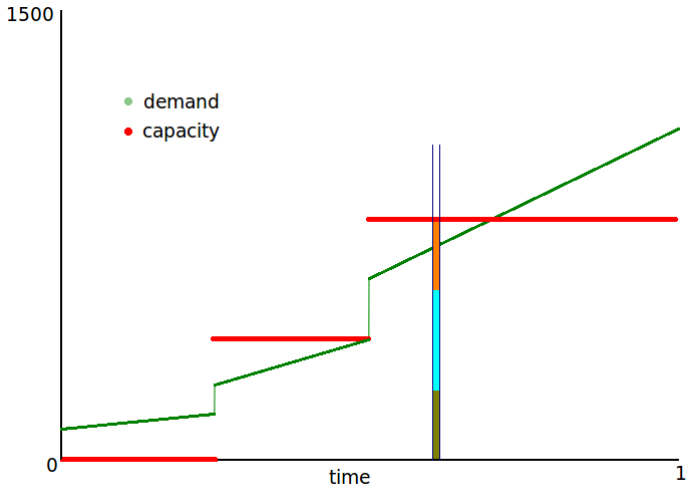
$x_1 = 0.25$ $x_2 = 0.5$
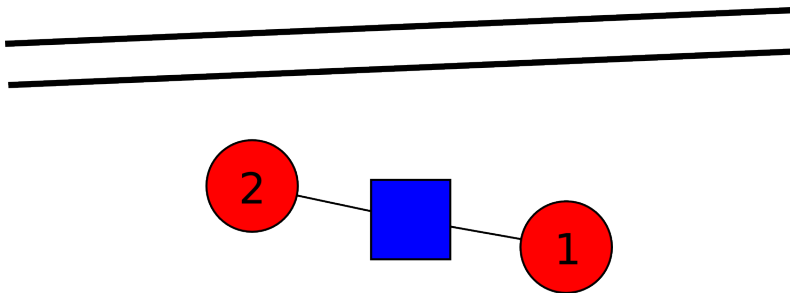
Example: Demand and Capacity

Evaluating the Objective: Brute Force

What to do in each interval?

Solve this assignment problem!

$$\min \sum_{i \in I, j \in J} c_{ij} y_{ij} + \sum_{j \in J} h q_j + \sum_{u \in N} o_u \beta_u + \sum_{u \in N} g \delta_u + \sum_{i \in I} z s_i$$

Subject to

$$s_i + \sum_{j \in J} y_{ij} \leq Cap_i$$
$$\sum_{i \in I} y_{ij} + q_j = D_j$$
$$\sum_{v:(u,v) \in E} \alpha_{uv} = -I_u + \delta_u + \beta_u + \sum_{i:m(i)=u} s_i$$
$$\alpha_{uv} = \frac{\theta_u - \theta_v}{b_{uv}}$$
$$-C_{uv} \leq \alpha_{uv} \leq C_{uv}$$
$$\beta_u \leq G_u$$

Evaluating the Objective: Main Idea

- Even though the variables are contnuous in time, the allocation of demands to open stations only changes at certain points in time, the breakpoints:
  - When a new station is built
  - When a station reaches its full capacity
  - When demand from a route reaches 0 at a station
- It is reasonably easy to find those breakpoints sequentially if the demand function is known
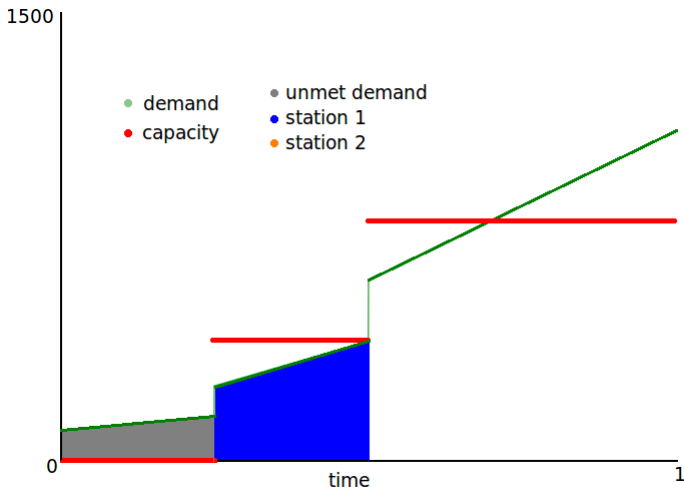
Example: t=0

Assign all demand to unmet demand
Next Breakpoint = 0.25, time when new station opens
Add 0 to objective
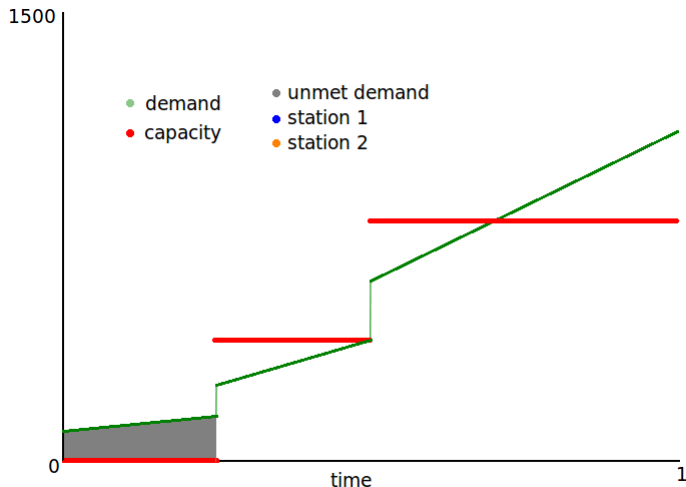
Example: t=0.25

Assign all
demand to
station 1
Next
breakpoint =
0.5, time when
station 1
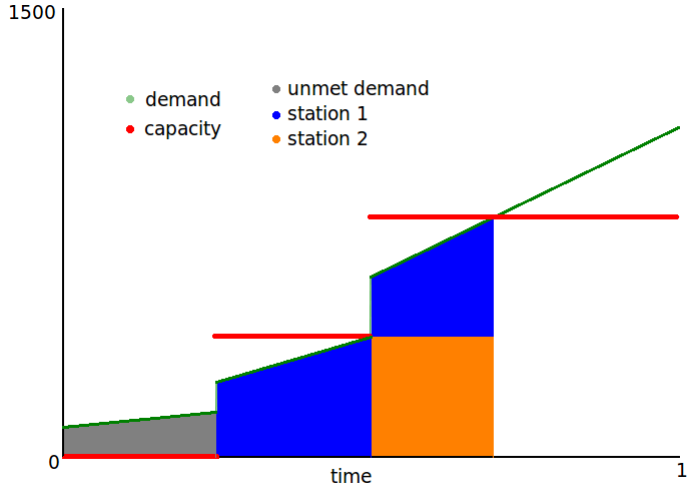reaches full
capacity
Add discounted
area to
objective

Dicounted Area Calculation

$$\int_c^d r^t(at+b)dt = \frac{r^c(a - \ln r(ac+b)) + r^d(-a + \ln r(ad+b))}{\ln r^2}$$

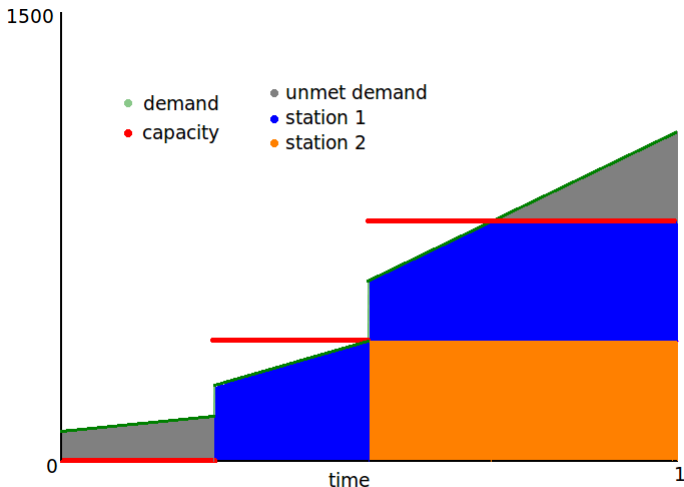$$\int_c^d r^t ab^t dt = \frac{a(b^d r^d - b^c r^c)}{\ln b + \ln r}$$

Example: t=0.5

Assign max
demand to
station 2, since
it is cheaper.
Assign the rest
to station 1
Next
breakpoint =
0.7, time when
station 1
reaches full
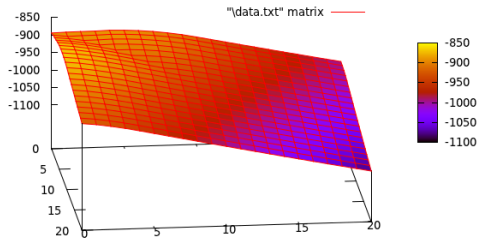capacity
Add discounted
area to
objective

Example: t=0.7

Assign full
demand to
stations 1 and
2, and the rest
to unmet
demand
Next
breakpoint = 1,
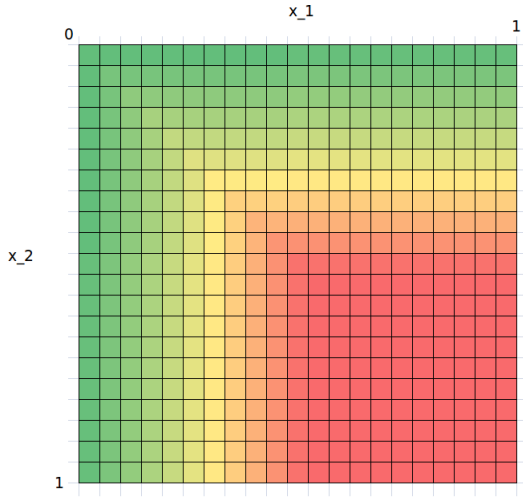end of horizon
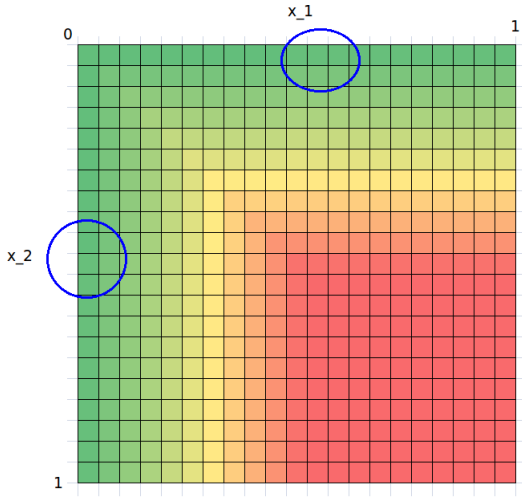Add discounted
area to
objective

Objective Function Example Visualization

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Objective Function Example Visualization

Optimization Method

- Projected gradient method
- $O(1)$ approximate gradient calculation as opposed to $O(n)$
- A heuristic for choosing initial point works well

What About Stochasticity?

- Uncertainty in next breakpoint calculation
- Can calculate expected cost between two breakpoints
- Can use mean or median or worst values for finding next breakpoint
- Can use single scenarios for gradient descent method, but use a full Monte Carlo evaluation at the final stopping point

Conclusions

- Can evaluate objective if demand function allows for:
    - Easy breakpoint search
    - Easy 'discounted' integration
- Finding minimum by a grid method is hard
- Can answer questions such as 'how much should the govt subsidize building of those stations so that PHEV demand increases by $x\%$

Demand Under Different Subsidy Levels
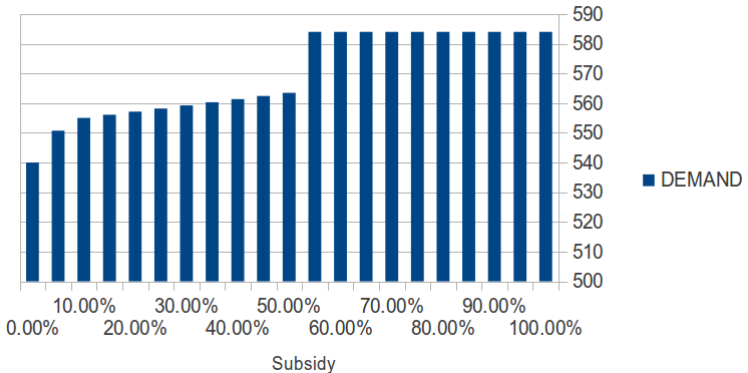
Future Work

- Develop a good optimization method for finding the minimum of the non-convex objective
- Test on an existing benchmark
- Incorporate stochasticity in demand
- Find computational compelxity of method